Ujian Tengah Semester

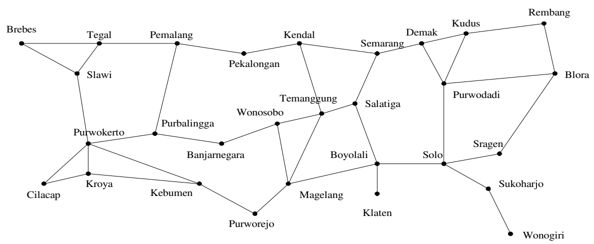
Sesi 08

Graph

Sesi 09

Pendahuluan

- Graf digunakan untuk merepresentasikan objek-objek diskrit dan hubungan antara objek-objek tersebut.
- Gambar di bawah ini sebuah graf yang menyatakan peta jaringan jalan raya yang menghubungkan sejumlah kota di Provinsi Jawa Tengah.

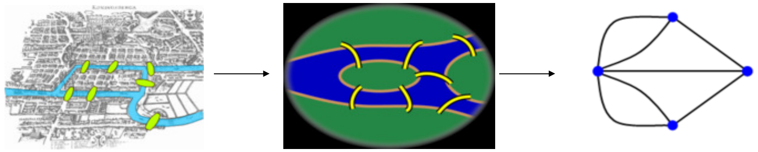


History

- Basic ideas were introduced in the eighteenth century by Leonard Euler (Swiss mathematician)
- Euler was interested in solving the Königsberg bridge problem (Town of Königsberg is in Kaliningrad, Republic of Russia)
- Graphs have several applications in many areas:
  - Study of the structure of the World Wide Web
  - Shortest path between 2 cities in a transportation network
  - Molecular chemistry

Graph Theory - History

Leonhard Euler's paper on "*Seven Bridges of Königsberg*",
published in 1736.

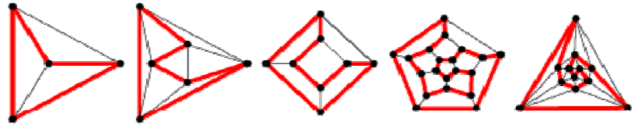


Graph Theory - History

Cycles in Polyhedra

Thomas P. Kirkman

William R. Hamilton

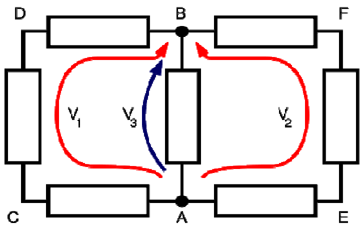


Hamiltonian cycles in Platonic graphs

Graph Theory - History

Trees in Electric Circuits

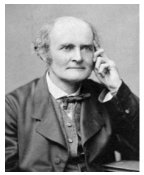
Gustav Kirchhoff



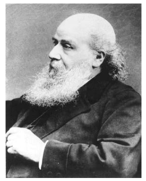
$V_1 = V_2 = V_3$

Graph Theory - History

Enumeration of Chemical Isomers

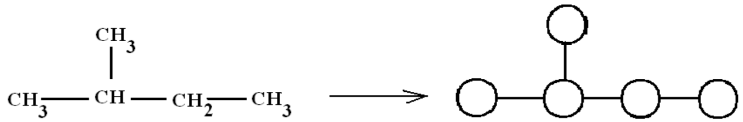


Arthur Cayley



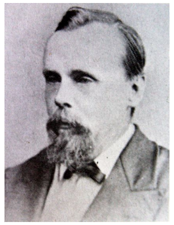
James J. Sylvester

George Polya

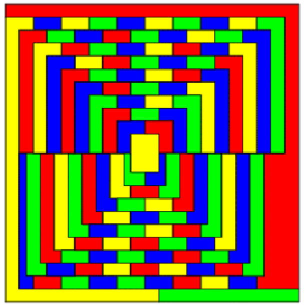


Graph Theory - History

Four Colors of Maps




Francis Guthrie Auguste DeMorgan



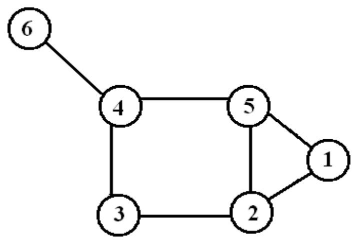
Definition: Graph

- G is an ordered triple G:=(V, E, f)
  - V is a set of nodes, points, or vertices.
  - E is a set, whose elements are known as edges or lines.
  - f is a function
    - maps each element of E
    - to an unordered pair of vertices in V.

Definitions

- Vertex
  - Basic Element
  - Drawn as a *node* or a *dot*.
  - **Vertex set** of *G* is usually denoted by *V*(*G*), or *V*
- Edge
  - A set of two elements
  - Drawn as a line connecting two vertices, called end vertices, or endpoints.
  - The edge set of G is usually denoted by E(G), or E.

Example



- V:={1,2,3,4,5,6}
- E:={{1,2},{1,5},{2,3},{2,5},{3,4},{4,5},{4,6}}

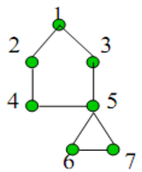
Definitions

A ***graph*** G comprises a set V of vertices and a set E of edges

Each ***edge*** in E is a pair (a,b) of vertices in V

If (a,b) is an edge in E, we connect a and b in the ***graph drawing*** of G

Example:



$V=\{1,2,3,4,5,6,7\}$

$E=\{(1,2),(1,3),(2,4),(4,5),(3,5),(4,5),(5,6),(6,7)\}$

5 main categories of graphs

1. Simple graph
2. Multigraph
3. Pseudograph
4. Directed graph
5. Weighted graphs
6. Directed multigraph

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Simple graph

- Definition 1

A simple graph G = (V,E) consists of V, a nonempty set of vertices, and E, a set of unordered pairs of distinct elements of V called edges.

- Example: Telephone lines connecting computers in different cities.

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Multigraph

- Definition 2:

A multigraph G = (V,E) consists of a set E of edges, and a function f from E to $\{\{u,v\} \mid u, v \in V, u \neq v\}$. The edges e_1 and e_2 are called multiple or parallel edges if $f(e_1) = f(e_2)$.

- Example: Multiple telephone lines connecting computers in different cities.

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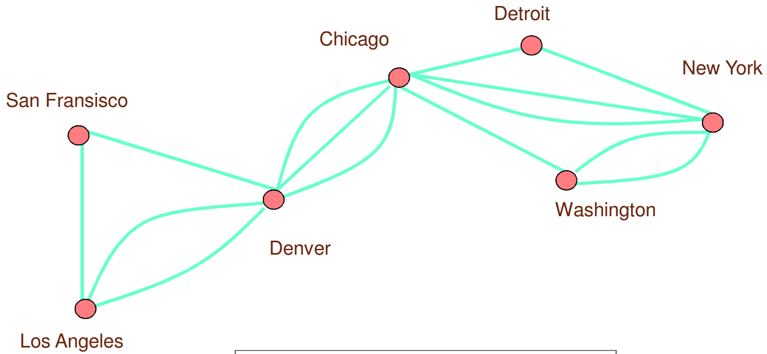
Ex Multigraph



A Computer network with multiple lines

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Pseudograph

- Definition 3:

A pseudograph G = (V,E) consists of a set V of vertices, a set E of edges, and a function f from E to {{u,v} | u, v ∈ V}. An edge is a loop if f(e) = {u,u} = {u} for some u ∈ V.

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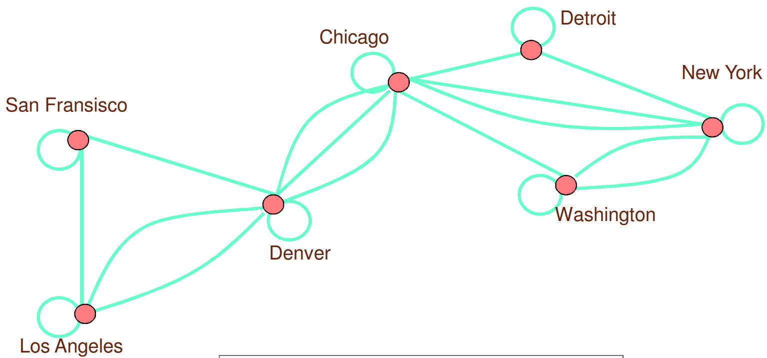
Ex Pseudograph



A Computer network with diagnostic lines

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Directed graph

- Definition 4:

A directed graph (V,E) consists of a set of vertices V and a set of edges E that are ordered pairs of elements of V.

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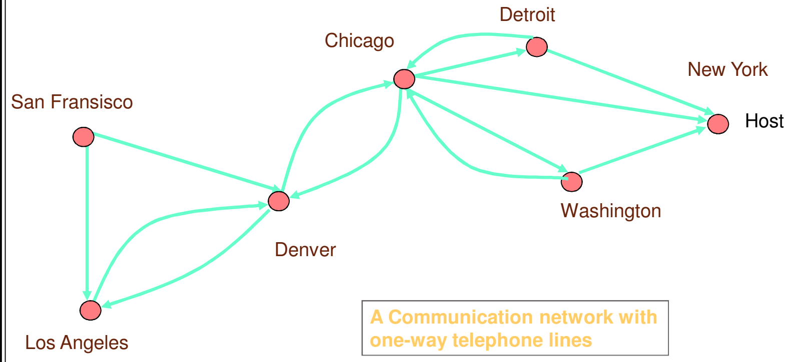
Ex Directed graph



A Communication network with one-way telephone lines

This example shows that the host computer <u>can only receive data</u> from other computer, it cannot emit

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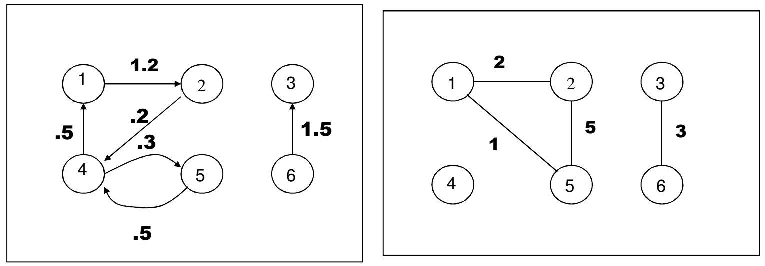
Weighted graphs

- is a graph for which each edge has an associated ***weight***, usually given by a ***weight function*** $w: E \to \mathbf{R}$.



Directed multigraph

- Definition 5:

A directed multigraph G = (V,E) consists of a set V of vertices, a set E of edges, and a function f from E to $\{\{u,v\} \mid u, v \in V\}$. The edges e_1 and e_2 are multiple edges if $f(e_1) = f(e_2)$.

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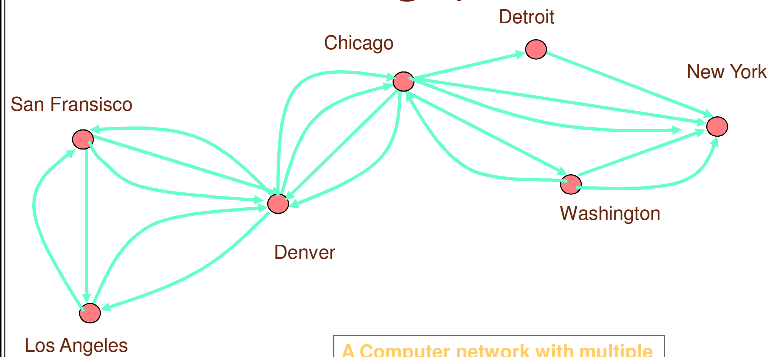
Ex Directed multigraph



A Computer network with multiple one-way telephone lines

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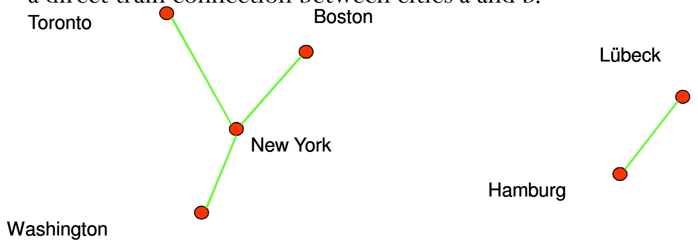
Graph Models

• **Example I:** How can we represent a network of (bi-directional) railways connecting a set of cities?

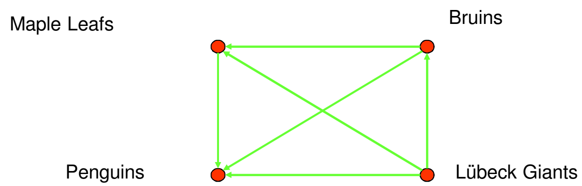
• We should use a **simple graph** with an edge {a, b} indicating a direct train connection between cities a and b.



Graph Models

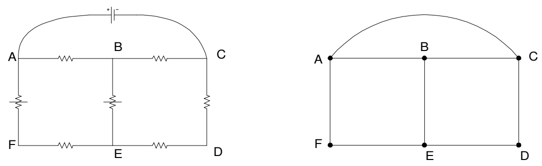
• **Example II:** In a round-robin tournament, each team plays against each other team exactly once. How can we represent the results of the tournament (which team beats which other team)?

• We should use a **directed graph** with an edge (a, b) indicating that team a beats team b.



Contoh Terapan Graf

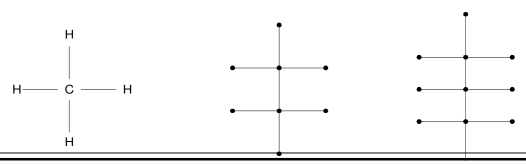
1. *Rangkaian listrik.*



(a) (b)

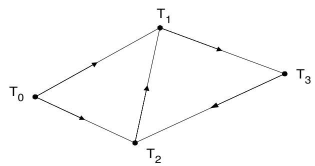
2. *Isomer senyawa kimia karbon*

metana (CH_4) etana (C_2H_6) propana (C_3H_8)



3. *Transaksi konkuren pada basis data terpusat*

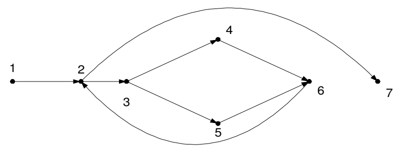
Transaksi T_0 menunggu transaksi T_1 dan T_2
Transaksi T_2 menunggu transaksi T_1
Transaksi T_1 menunggu transaksi T_3
Transaksi T_3 menunggu transaksi T_2



deadlock!

4. *Pengujian program*

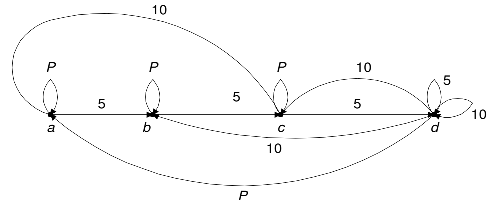
```
read(x);
while x <> 9999 do
 begin
   if x < 0 then
      writeln('Masukan tidak boleh negatif')
   else
     x:=x+10;
   read(x);
 end;
writeln(x);
```



Keterangan:
1 : read(x)
2 : x <> 9999
3 : x < 0
4 : writeln('Masukan tidak boleh negatif');
5 : x := x + 10
6 : read(x)
7 : writeln(x)

5. *Terapan graf pada teori otomata* [LIU85].

Mesin jaja (*vending machine*)



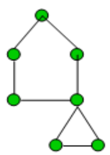
Keterangan:
a : 0 sen dimasukkan
b : 5 sen dimasukkan
c : 10 sen dimasukkan
d : 15 sen atau lebih dimasukkan

Size Order

The ***size*** of *G* is the number *n* of vertices in *V*

The ***order*** of *G* is the number *L* of edges in *E*

Minimum possible order is 0 (***empty*** graph)
Maximum possible order is $n(n-1)/2$ (***complete*** graph)

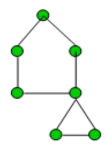


Size = 7, Order = 8

Density

The ***density*** of *G* is the ratio of edges in *G* to the maximum possible number of edges

$$\text{Density} = \frac{2L}{n(n-1)}$$



Density = 2×8/(7×6) = 8/21

Graph Terminology

- **Definition:** Two vertices u and v in an undirected graph G are called **adjacent** (or **neighbors**) in G if {u, v} is an edge in G.
- If e = {u, v}, the edge e is called **incident with** the vertices u and v. The edge e is also said to **connect** u and v.
- The vertices u and v are called **endpoints** of the edge {u, v}.

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Graph Terminology

- **Definition:** The **degree** of a vertex in an undirected graph is the number of edges incident with it, except that a loop at a vertex contributes twice to the degree of that vertex.
- In other words, you can determine the degree of a vertex in a displayed graph by **counting the lines** that touch it.
- The degree of the vertex v is denoted by **deg(v)**.

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Graph Terminology

- A vertex of degree 0 is called **isolated**, since it is not adjacent to any vertex.
- **Note:** A vertex with a **loop** at it has at least degree 2 and, by definition, is **not isolated**, even if it is not adjacent to any **other** vertex.
- A vertex of degree 1 is called **pendant**. It is adjacent to exactly one other vertex.

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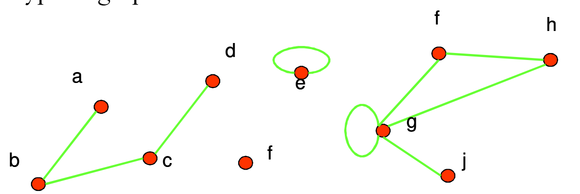
Graph Terminology

- **Example:** Which vertices in the following graph are isolated, which are pendant, and what is the maximum degree? What type of graph is it?



Solution: Vertex f is isolated, and vertices a, d and j are pendant. The maximum degree is deg(g) = 5.

This graph is a pseudograph (undirected, loops).

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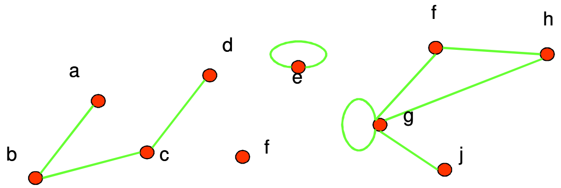
Graph Terminology

• Let us look at the same graph again and determine the number of its edges and the sum of the degrees of all its vertices:



Result: There are 9 edges, and the sum of all degrees is 18. This is easy to explain: Each new edge increases the sum of degrees by exactly two.

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Graph Terminology

• **The Handshaking Theorem:** Let G = (V, E) be an undirected graph with e edges. Then

$$2e = \sum_{v \in V} deg(v).$$

• **Example:** How many edges are there in a graph with 10 vertices, each of degree 6?

• **Solution:** The sum of the degrees of the vertices is 6·10 = 60. According to the Handshaking Theorem, it follows that 2e = 60, so there are 30 edges.

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Graph Terminology

• **Definition:** When (u, v) is an edge of the graph G with directed edges, u is said to be **adjacent to** v, and v is said to be **adjacent from** u.

• The vertex u is called the **initial vertex** of (u, v), and v is called the **terminal vertex** of (u, v).

• The initial vertex and terminal vertex of a loop are the same.

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Graph Terminology

• **Definition:** In a graph with directed edges, the **in-degree** of a vertex v, denoted by **$deg^-(v)$**, is the number of edges with v as their **terminal vertex**.

• The **out-degree** of v, denoted by **$deg^+(v)$**, is the number of edges with v as their initial vertex.

• **Question:** How does adding a loop to a vertex change the in-degree and out-degree of that vertex?

• **Answer:** It increases both the in-degree and the out-degree by one.

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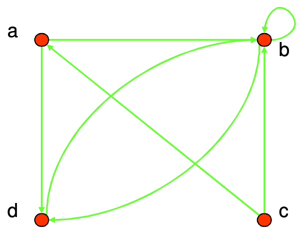
Graph Terminology

• **Example:** What are the in-degrees and out-degrees of the vertices a, b, c, d in this graph:

$\deg^-(a) = 1$
$\deg^+(a) = 2$

$\deg^-(b) = 4$
$\deg^+(b) = 2$



$\deg^-(d) = 2$
$\deg^+(d) = 1$

$\deg^-(c) = 0$
$\deg^+(c) = 2$

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Graph Terminology

• **Theorem:** Let G = (V, E) be a graph with directed edges. Then:

$$\sum_{v \in V} \deg^-(v) = \sum_{v \in V} \deg^+(v) = |E|$$

• This is easy to see, because every new edge increases both the sum of in-degrees and the sum of out-degrees by one.

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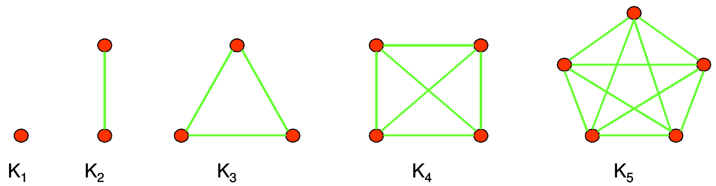
Special Graphs

• **Definition:** The **complete graph** on n vertices, denoted by K_n, is the simple graph that contains exactly one edge between each pair of distinct vertices.



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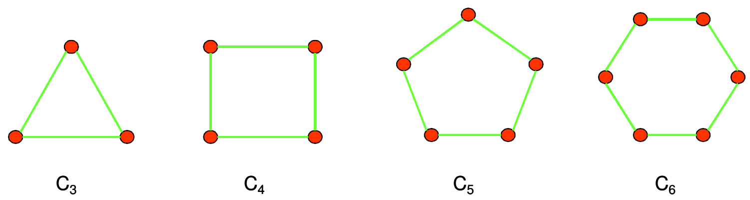
Special Graphs

• **Definition:** The **cycle** C_n, $n \geq 3$, consists of n vertices $v_1, v_2, \ldots, v_n$ and edges $\{v_1, v_2\}, \{v_2, v_3\}, \ldots, \{v_{n-1}, v_n\}, \{v_n, v_1\}$.



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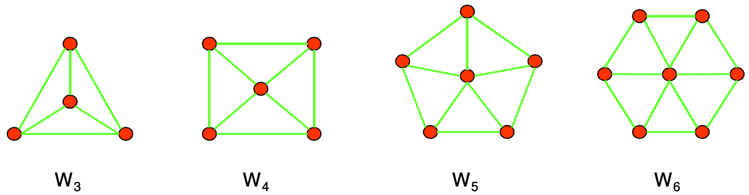
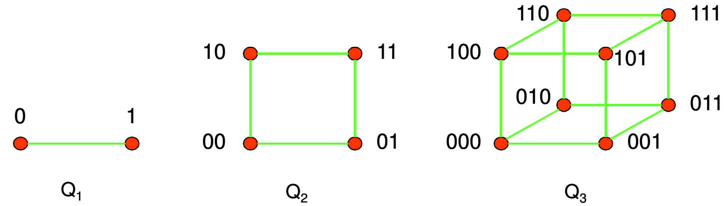
Special Graphs

- **Definition:** We obtain the **wheel** W_n when we add an additional vertex to the cycle C_n, for $n \geq 3$, and connect this new vertex to each of the n vertices in C_n by adding new edges.



Special Graphs

- **Definition:** The **n-cube,** denoted by Q_n, is the graph that has vertices representing the 2^n bit strings of length n. Two vertices are adjacent if and only if the bit strings that they represent differ in exactly one bit position.

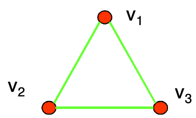


Special Graphs

- **Definition:** A simple graph is called **bipartite** if its vertex set V can be partitioned into two disjoint nonempty sets V_1 and V_2 such that every edge in the graph connects a vertex in V_1 with a vertex in V_2 (so that no edge in G connects either two vertices in V_1 or two vertices in V_2).
- For example, consider a graph that represents each person in a village by a vertex and each marriage by an edge.
- This graph is **bipartite**, because each edge connects a vertex in the **subset of males** with a vertex in the **subset of females** (if we think of traditional marriages).

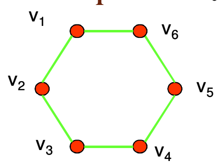
Special Graphs

- **Example I:** Is C_3 bipartite?

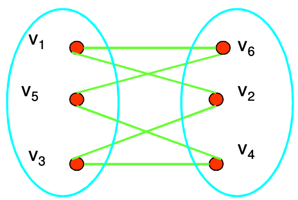


No, because there is no way to partition the vertices into two sets so that there are no edges with both endpoints in the same set.

- **Example II:** Is C_6 bipartite?

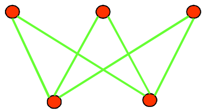


Yes, because we can display C_6 like this:

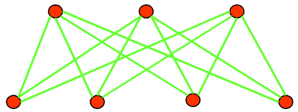


Special Graphs

•**Definition:** The **complete bipartite graph** $K_{m,n}$ is the graph that has its vertex set partitioned into two subsets of m and n vertices, respectively. Two vertices are connected if and only if they are in different subsets.



$K_{3,2}$



$K_{3,4}$

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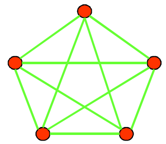
Operations on Graphs

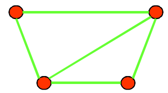
•**Definition:** A **subgraph** of a graph G = (V, E) is a graph H = (W, F) where $W \subseteq V$ and $F \subseteq E$.

•**Note:** Of course, H is a valid graph, so we cannot remove any endpoints of remaining edges when creating H.

•**Example:**



K_5



subgraph of K_5

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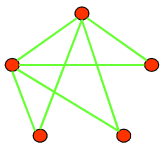
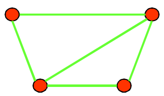
Operations on Graphs

•**Definition:** The **union** of two simple graphs $G_1 = (V_1, E_1)$ and $G_2 = (V_2, E_2)$ is the simple graph with vertex set $V_1 \cup V_2$ and edge set $E_1 \cup E_2$.

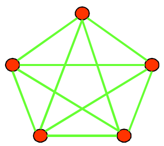
•The union of G_1 and G_2 is denoted by $\mathbf{G_1 \cup G_2}$.



G_1



G_2



$G_1 \cup G_2 = K_5$

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